Topological phases of quantum matter

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2016 Nobel prize in physics awarded to David Thouless, Duncane Haldane and Michael Kosterlitz

for theoretical discoveries of topological phase transitions and topological phases of matter

Although first to mention topological phases, not really the first Nobel prize for topological phases

First - 1982 Nobel to Klaus von Klitzing for `discovery of quantised Hall effect '

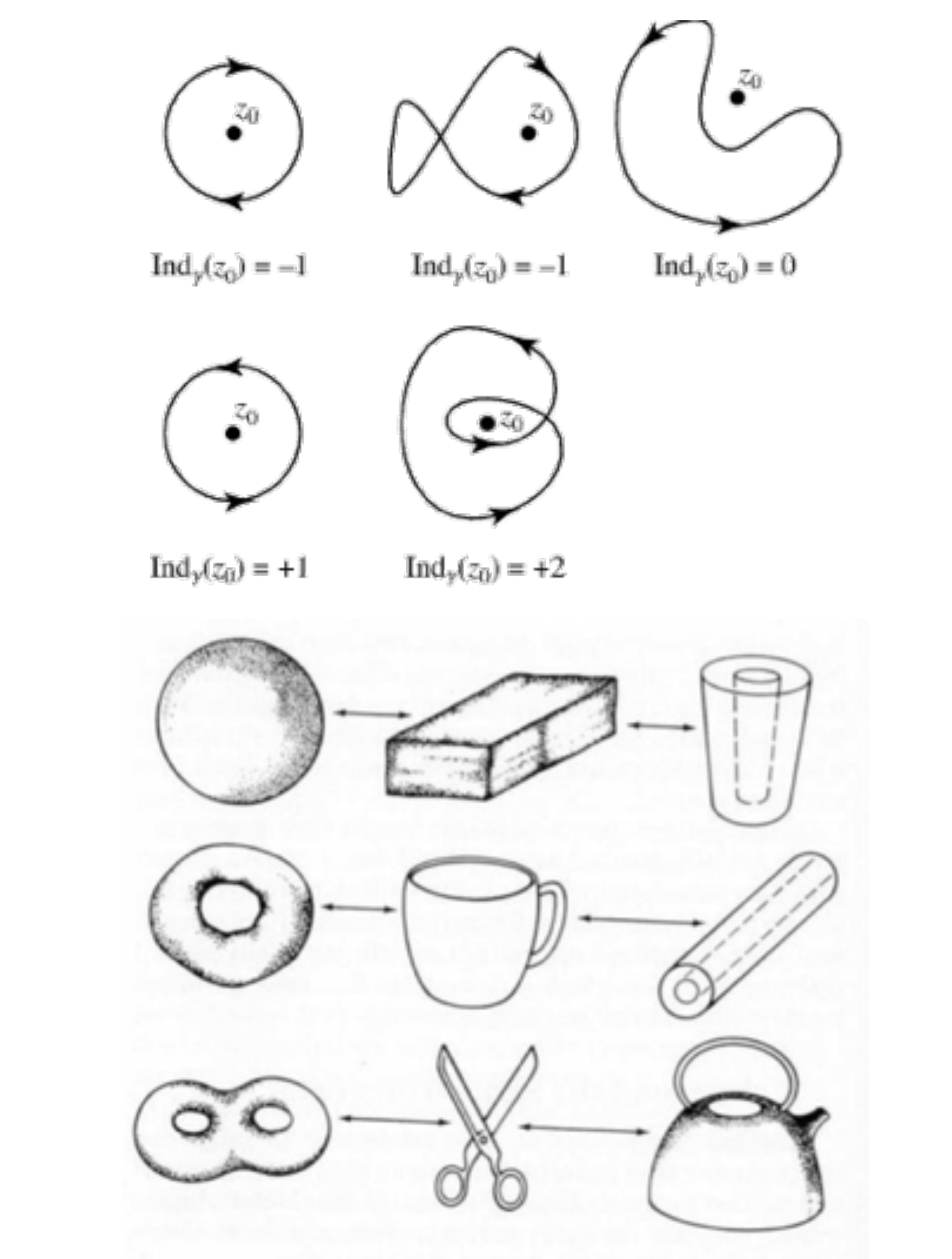
Second - 1998 Nobel to Robert Laughlin, Horst Stormer and Daniel Tsui for `discovery of a new form of quantum fluid with fractionally charged excitations'

Expect many more in the future

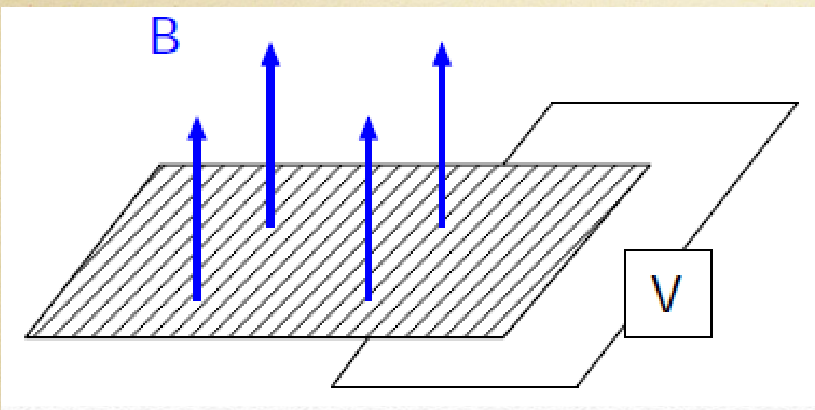
What do we mean by phases?

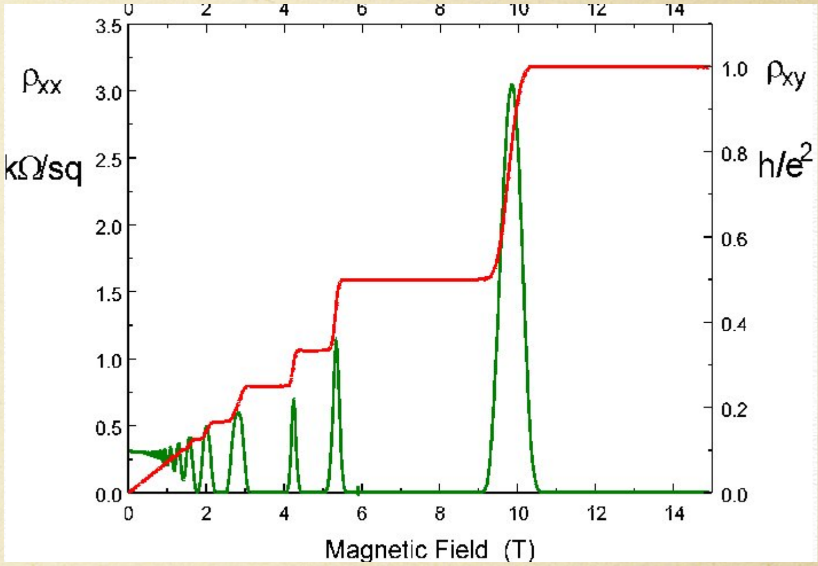
- Would have heard of solid, liquid, and gas phases
- But many more complicated phases exist. Can think of a phase as a collection of particles with some properties which distinguish them from other phases
- Earlier, phases were classified in terms of expectation values of `local' order parameters and broken symmetries -for example ferromagnets break rotational symmetry and have all spins pointing in one direction

- In last 10 years or so, a paradigm shift in the way of classifying phases - based on topology.
- Topology is a branch of mathematics that deals with quantities that are invariant under small continuous changes.
- Topological invariants such as winding numbers, genus of surfaces, etc, depend on the system as whole and cannot be determined locally
- Changing topological invariants requires drastic measures such as punching a hole!



Topological phases 30 years ago - the integer quantum Hall state



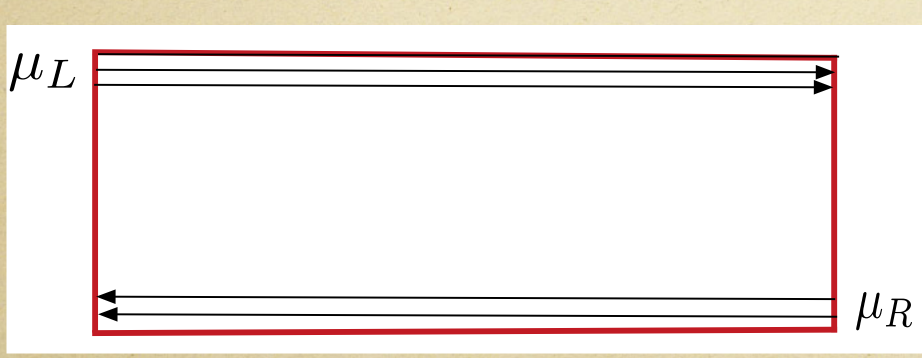


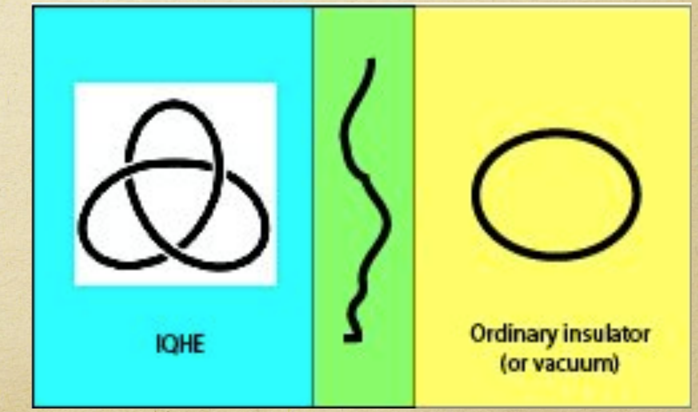
- Electrons constrained to move in two dimensions, in the presence of a magnetic field in the perpendicular direction showed quantised Hall conductance
- Happens because the bulk states in the middle of the sample are `gapped' (need energy to be excited) and do not carry current and only the states at the edges are `gapless' and carry current

- Thouless (and collaborators) understood that the quantum Hall state was an example of a topological phase.

Thouless, Kohmoto, Nightingale and den Nijs

- Realised that the wave functions get knotted by the magnetic field and form a phase which is distinct from other phases and can only change when drastic changes are made, such as when a gap is closed or number of edge states change





Current interest in topological insulators

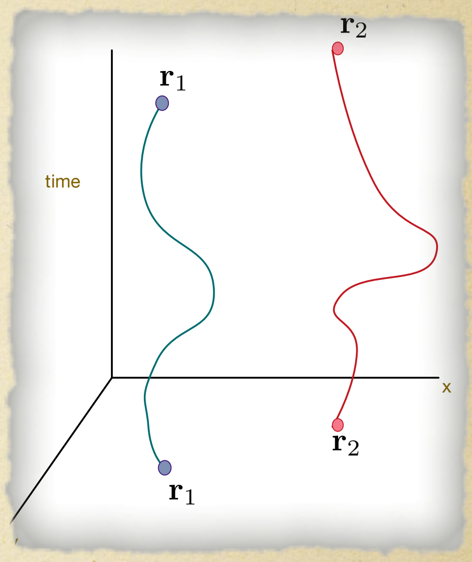
- In 2005, it was realised that the knotting of the wave-functions and the topologically quantised conductance can happen even in the absence of strong magnetic fields - need strong spin-orbit coupling instead
- So materials with strong spin-orbit coupling can `behave' like a quantum Hall state - insulating in the bulk and having conducting edge states - rather like a block of wood covered with a metal, except that it is made of a single material - these are the topological insulators

- Not discovered earlier for many reasons, including the need for sophisticated instruments that can map local density of states to know which states carry current
- Theoretically required understanding of Berry phase and structures of mappings of Brillouin zone to the Hamiltonian - more information than just the dispersion
- Predicted theoretically and then experimentally observed!

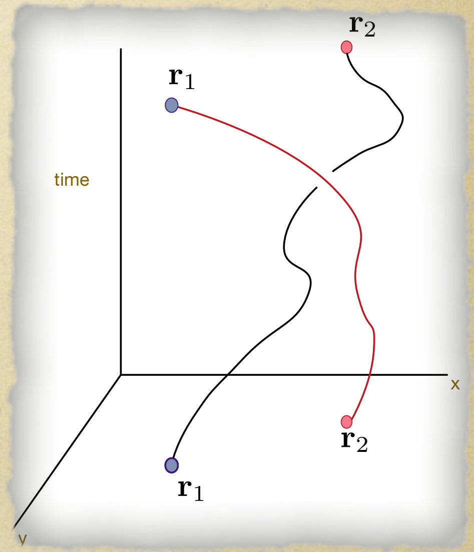
Haldane, Kane, Mele, Fu, Bernevig, Zhang, Molenkaamps, Konig, ...
Moore, Balents, Roy, Hassan,....

Topotronics or transport in topological phases

- Transport can be through quasiparticles (emergent excitations inside the material) which are not just bosons or fermions, but also anyons and even non-abelian quasiparticles like Majorana zero modes



- What does one mean by this? In quantum theory, the probability for a process is the square of the amplitude of its wave function. So we need to compute the total amplitude for indistinguishable particles to go from some position to another where they are `exchanged'. These are the world lines of the quasiparticles



- Quantum statistics actually reflects the topology of the world lines. Rules for how we add different contributions from different classes.

- In 3 dimensions, can only add or subtract leading to bosons and fermions

- But in 2 dimensions, can add the paths with any phase leading to anyon statistics (because removing a single point in 2D makes the space multiply connected)

- Turns out that in many of the topological phases, in particular, the fractional quantum Hall phase, the quasiparticle excitations with fractional charge turn out to be anyons.

Leinaas and Myrrheim, Wilczek

Non-abelian anyons

- Some topological phases also have non-abelian excitations, whose exchanges can give more than just a phase
- These particles have an internal degree of freedom which encodes a topological (no local measurement can distinguish between the states) degeneracy (more than one state with the same energy)
- So when these particles (with multicomponent wavefunctions) are exchanged, they also get rotated within this space. So the quantum state changes under exchange of identical particles
- Since rotations do not commute, these are called non-abelian anyons

Moore and Seiberg, Witten
Moore and Reed

Why all the excitement about topological phases?

- Possible applications to topological quantum computation.
- The non-abelian excitations imply a robust degeneracy of the ground state. The exchange or braiding of these excitations move between the degenerate ground states, but is insensitive to details such as noise or interactions. Hence could be useful in quantum computation.

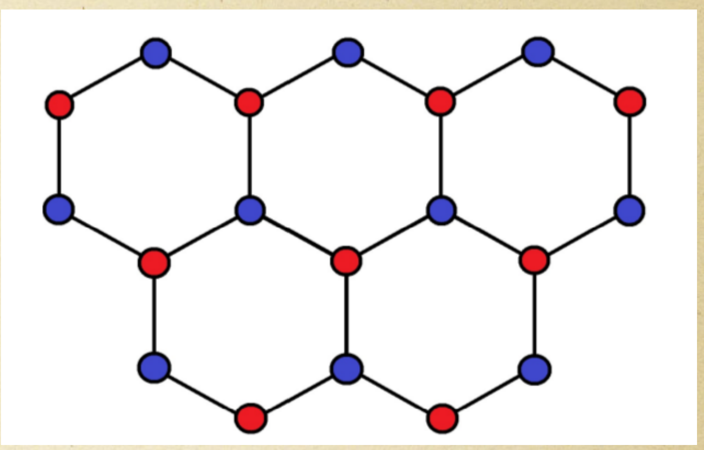
Kitaev, Preskill, Alicea, ...

Emergent Relativistic excitations

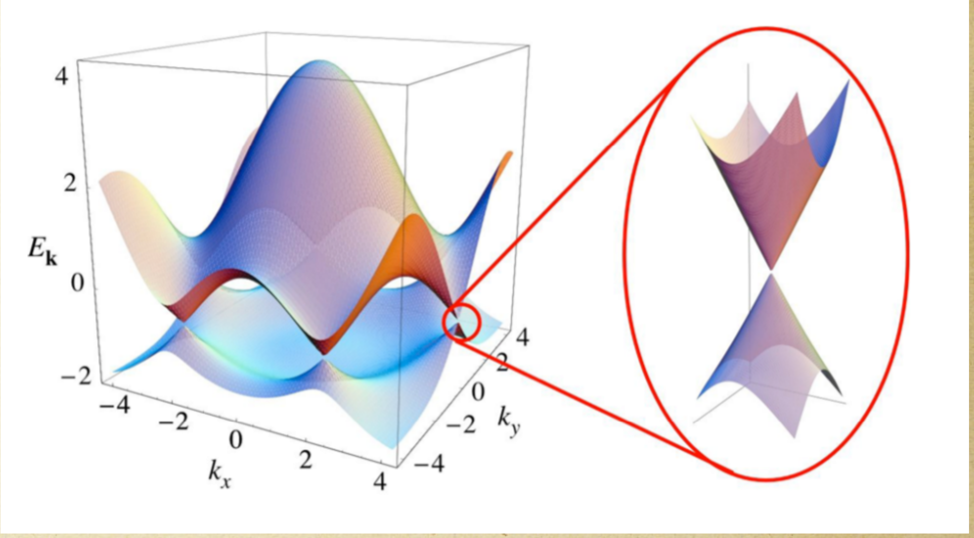
- Second paradigm shift - relativistic condensed matter physics
- Standard wisdom - electrons in solids are `low energy particles' and move at non-relativistic speeds and obey Schrodinger equation
- But recent realisation that in some cases, electrons in solids mathematically obey relativistic equations. Only the speed of light is replaced by the speed of the electron in the solid - the Fermi velocity
- First seen in 1D physics - quantum wires or quantum Hall edge states

Tomonaga, Luttinger

- Typical example in 2D - graphene - carbon atoms on a hexagonal lattice



- At low energies, dispersion is linear and electrons obey massless Dirac equation - only difference is that speed of light is now $v = 10^6 m/\mathrm{sec}$ and spin is replaced by pseudo spin (sub-lattice index)



Geim and Novoselov

Gapless topological phases

- Earlier understanding was that the bulk of the material should be insulating and only the surface states could be conducting for the topological phase to be stable
- But last few years have also given rise to metallic phases which are topological - the Weyl semi-metal phase
- How does this happen? Not so easy to see!

S. Murakami; Wan, Turner, Vishwanath and Savrasov
Xu *et al*; Lv *et al*

- There are other unusual technical features about the structure of the states of the Weyl semimetal in momentum space, and of the surface states, which are very different from usual metals and insulators
- This gives rise to interesting physical consequences. When magnetic field is applied in the same direction as an electric field, the resistance decreases with increasing magnetic field. This is not what happens in normal metals
- Are there other physical consequences? Perhaps! Part of current day research

Glimpses of my work in the field

1985

- Topological field theories now used as one of the tools to study topological phases. Worked on topological field theories - in particular, on non-abelian Chern Simons theory in 2+1 dimensions
- We obtained a topological Ward identity which showed that the coefficient of the non-abelian Chern-Simons term, which has to be quantised as an integer for topological reasons, gets renormalised only at one loop level by an integer and there are no further corrections beyond that. Surprise was that perturbation theory knew about topology!

Robert Pisarski and S.R, PRD32,2081

- A non-perturbative proof later given by Witten, who also connected the Chern-Simons theory with Jones polynomials, knots, etc.

Intermediate years

- Topological field theories and Chern Simons theories naturally led to the quantum Hall problem described by an abelian Chern Simons theory
- Led to the fractional quantum Hall effect problem and hence to their excitations which were anyons
- Also worked on edge states of the quantum Hall problem, which formed chiral (moving in one direction) Luttinger liquids (a name for interacting many-body systems in one dimension) and on quantum wires (non-chiral Luttinger liquids). Another area of interest has been graphene,

Main Collaborator for many years: Diptiman Sen

Ranjan Ghosh,Jainendra Jain, Dileep Jatkar, Dattu Gaitonde, P K Mohanty
Durganandini,Chitra,Siddhartha Lal,Ravichandra,Amit Agarwal,Abhiram Soori

Current work

- Part of the global effort to understand these topological phases theoretically
- Efforts to classify them, efforts to find newer phases, efforts to see how they behave when juxtaposed with other materials topological and non-topological, efforts to see what happens when these phases are driven, efforts to understand bulk-edge correspondence, efforts to see what happens when you include disorder, interactions, finite temperature, how much of any effect is purely topological and how much is not, efforts to understand how the excitations can lead to fault tolerant quantum computation, etc

Collaborators

PhD students

1) Udit Khanna
2) Aditya Banerjee
3) Ruchi Saxena
4) Dibyakanti Mukherjee
5) Krashna Mohan Tripathi

Postdoc

1) Priyanka Mohan

Former Students

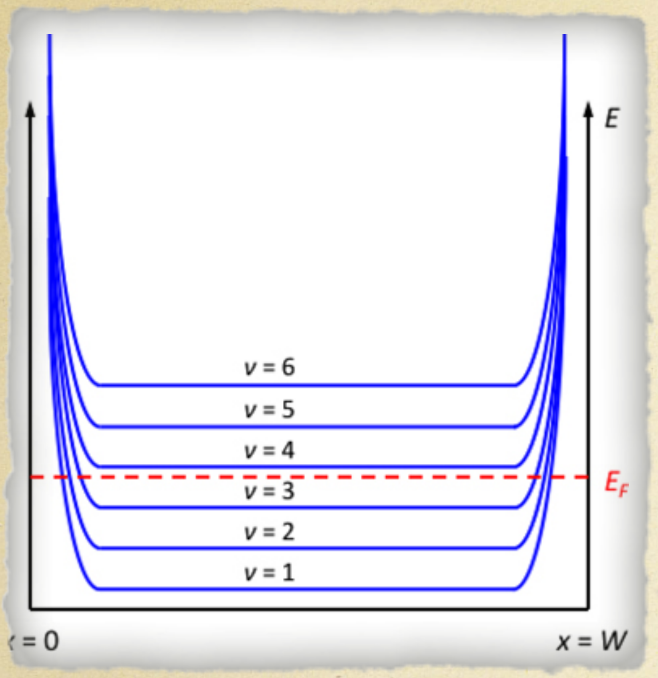
1) Sourin Das (faculty at IISER, Kolkata)
2) Arijit Saha (faculty at IOP, Bhubaneswar)
3) Arijit Kundu (faculty at IIT Kanpur)

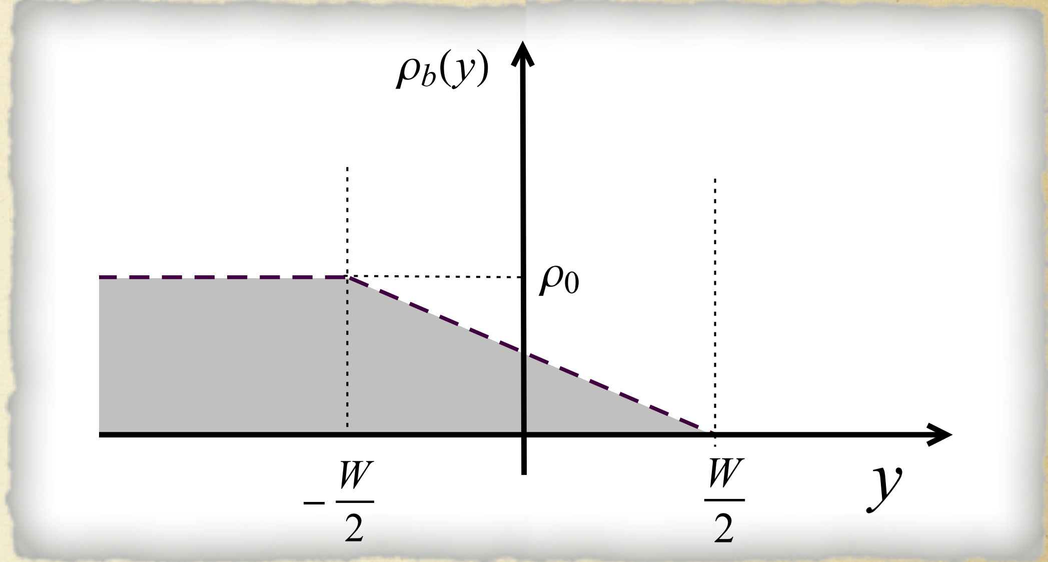
Master's Student

1) Namit Anand

1) Yuval Gefen (Weizmann Institute)
2) Ganapathy Murthy (Kentucky University)

Edge structure of integer quantum Hall effect

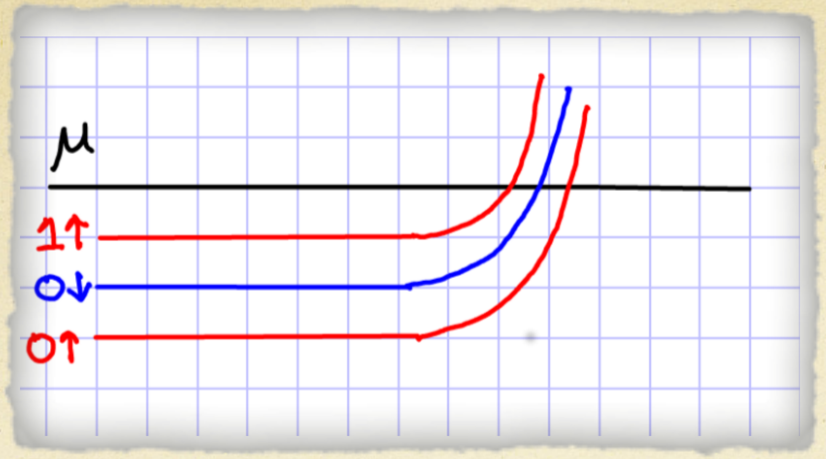




- The integer quantum Hall effect in the bulk is well understood without interactions and is known to have a conductance given by $G = \nu e^2/h$ for ν filled Landau levels

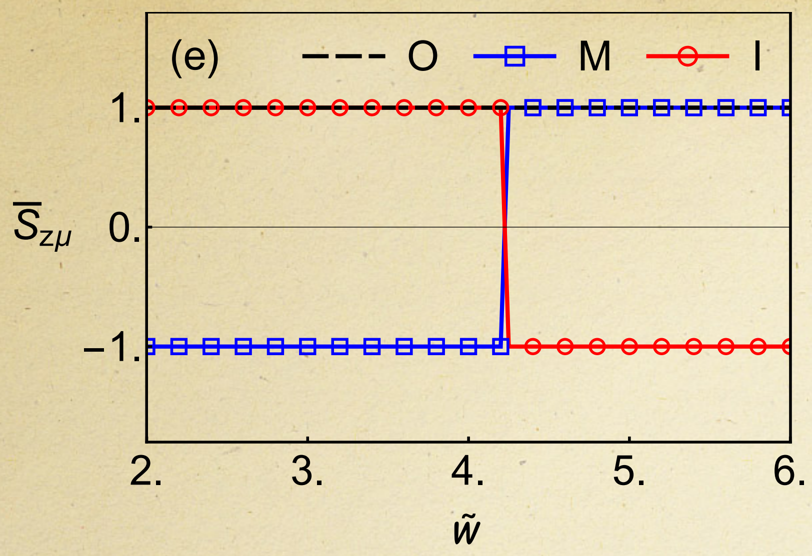
- Naively, this means that at the edges also, there should be ν massless chiral edge states with opposite chirality at the two edges

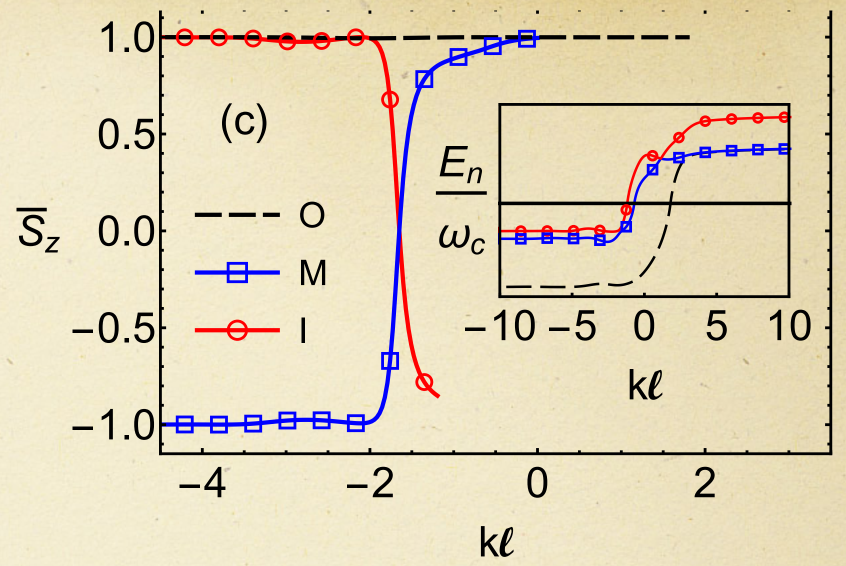
- But at the edges, it has been known that the Coulomb interactions between the electrons play a role
- Depending on whether the potential at the edge of the quantum Hall sample is abrupt or smooth, the number and position of the quantum Hall edge states can be changed - called charge reconstruction   Dempsey and Halperin, Chamon and Wen
- Essentially due to the fact that the state lowers its energy at the edge by reconstructing the Landau levels so as to spread out the charge density to follow the background density



- Question we asked : If there are multiple edge modes with the same spin, then is an exchange interaction driven rearrangement possible?
- The simplest case here is to have $\nu = 3$ because that will have at least 2 modes with the same spins
- For very sharp edges, the 3 modes are close to each other, but as the potential is made smoother, they move away from each other

Udit Khanna, Ganpathy Murthy, S.R and Yuval Gefen





- At some point, they are far enough so that they can gain energy by having two like spins next to each other, even making up for the loss in energy in flipping the spins

- Here the spins of the two inner modes have flipped so that they have the same spin

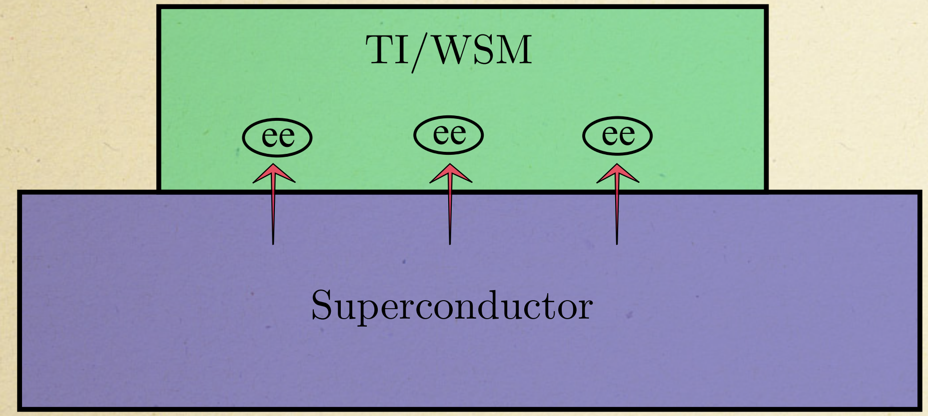
$$(0\uparrow, 0\downarrow, 1\uparrow) \longrightarrow (0\uparrow, 0\uparrow, 1\downarrow)$$

- For other parameters, the spins of the two outer modes switch

Work on Weyl semimetals

- We have also been working on Weyl semimetals for a while now
- Since it is know that when topological insulators are kept close to superconductors, new phenomena such as topological superconductor and new excitations with non-abelian statistics arise, we focussed on what happens when Weyl semimetals are juxtaposed with a superconductor
- We also found that the experiments to detect exotic effects become more feasible when some of the parameters can be changed externally. We found that shining light on the Weyl semimetals is an easy way to change some relevant parameters. So we focussed on that as well.

Proximity induced superconductivity in Weyl semi-metals

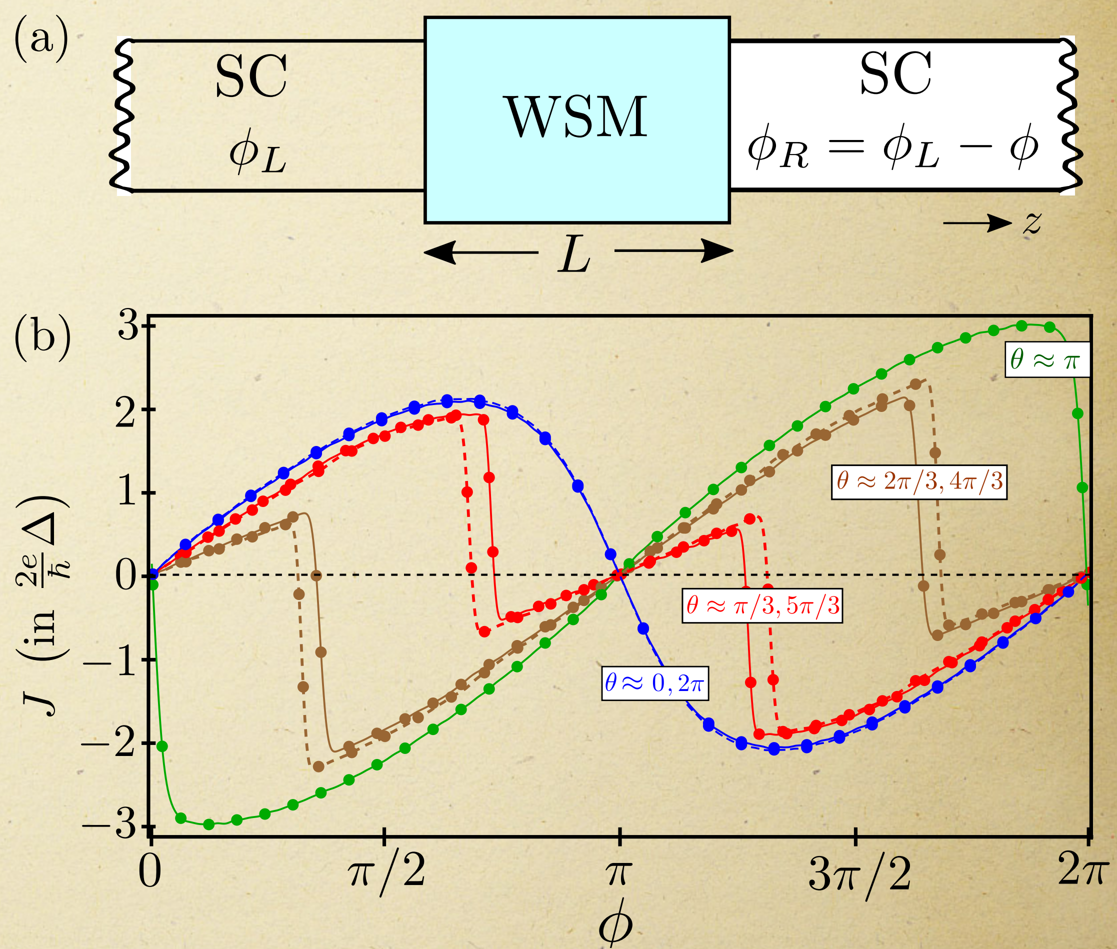


Udit Khanna, Arijit Kundu, Saurabh Pradhan and S.R., PRB (2014)

- We coupled one side of the topological insulator or Weyl semi-metals to a superconductor. Our aim was to see how much the superconducting correlations survive inside the TI/WSM.
- We found that it does not penetrate very much even for the WSM, unlike in metals. Also, since the Weyl nodes are chiral, this correlation did not give rise to a mass gap at the node.

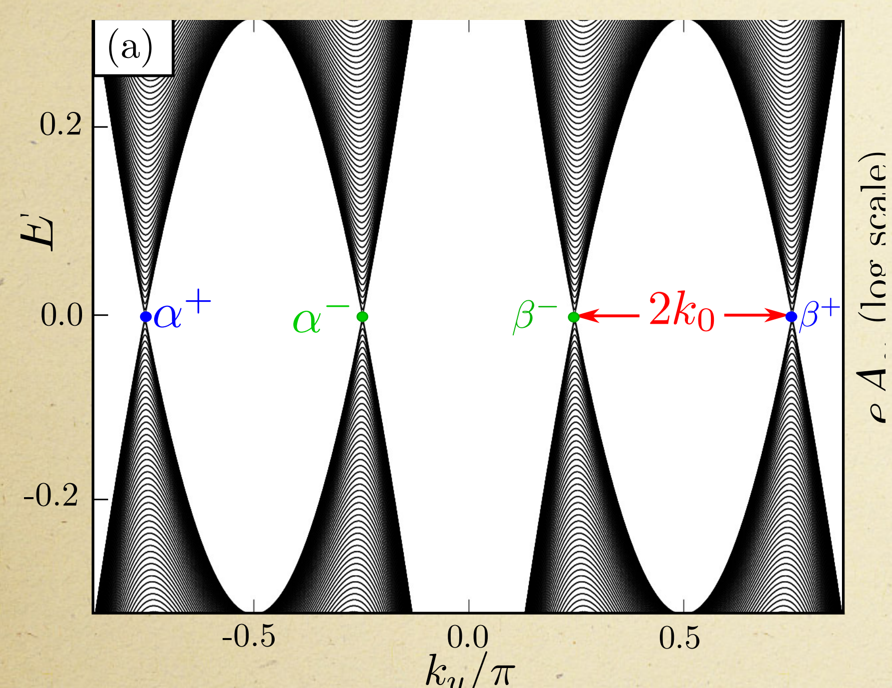
Josephson effect in Weyl semi-metals

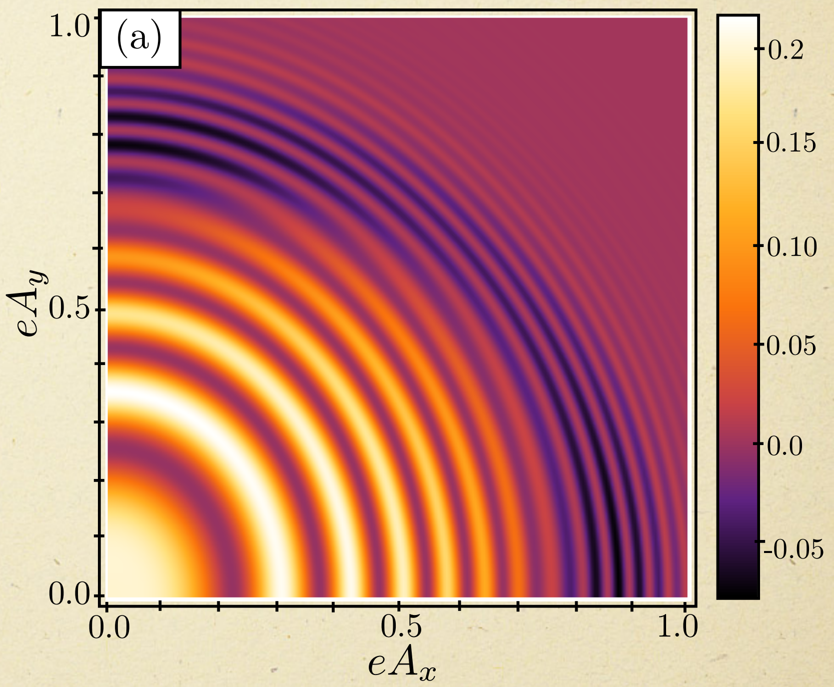
- We studied Josephson effect in Superconductor-Weyl semimetal-Superconductor junctions and found that the Josephson current is periodic in length with period π/k_0 where k_0 was the new scale induced by time-reversal breaking in the WSM
- The oscillations changed the sign of the critical current - i.e., there were periodic $0-\pi$ transitions



Udit Khanna, Dibyakanti Mukherjee, Arijit Kundu and S.R, PRB 93, 121409(R), 2017

Tuning transitions by irradiation





- The effective value of k_0 can be tuned by shining light on the Weyl semimetal

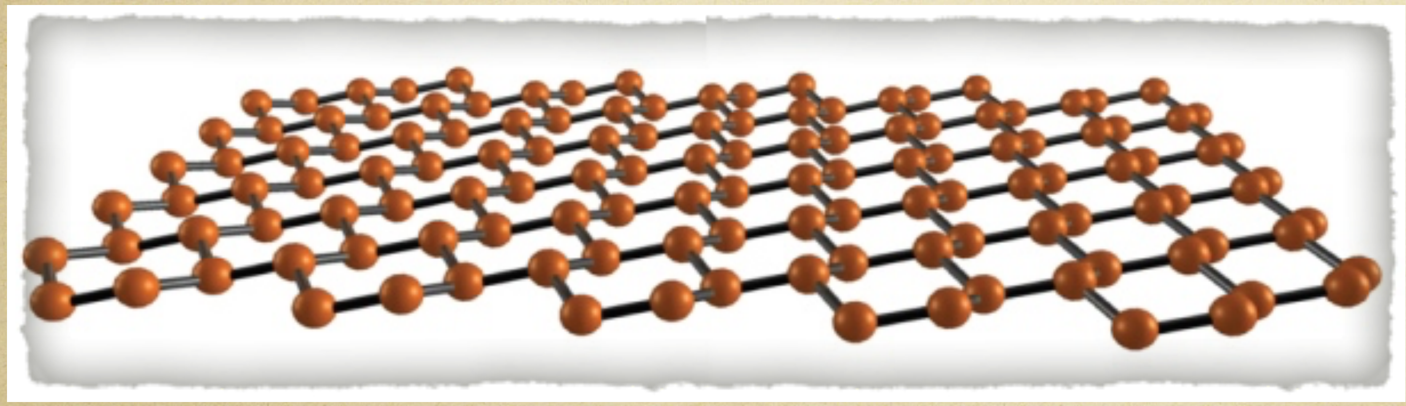
Udit Khanna, S.R and Arijit Kundu, PRB 95, 201115(R), 2017

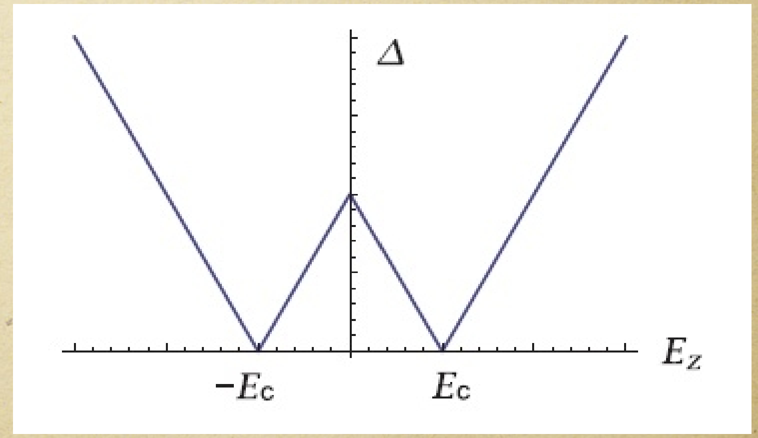
- This makes the $0 - \pi$ transition experimentally accessible

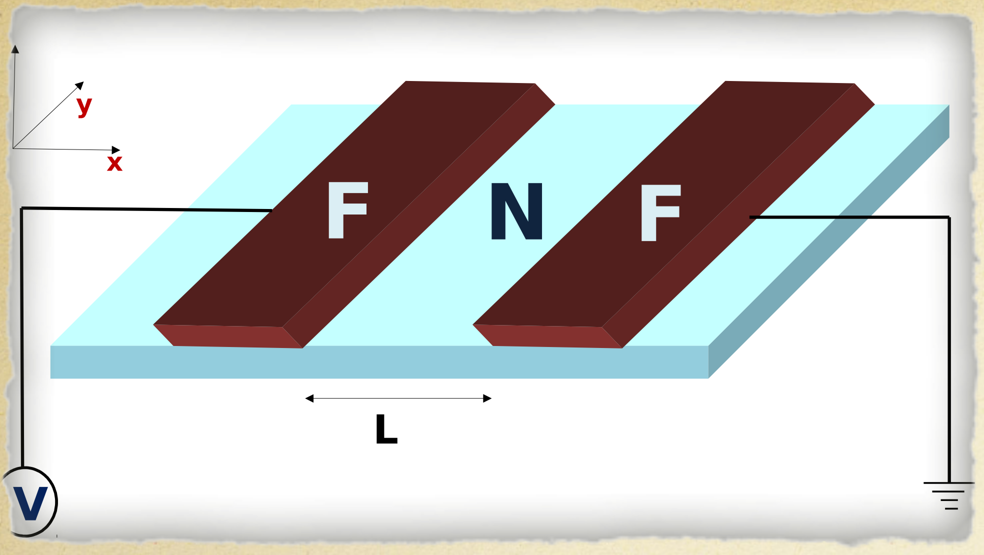
- We have also been working on other materials like silicene (similar to graphene) which has a buckled structure, and in general, materials with larger spin-orbit coupling than graphene

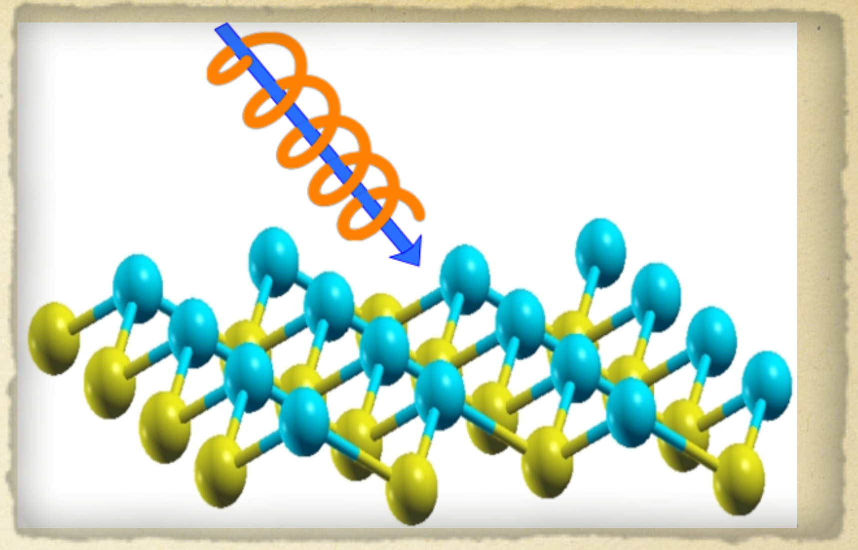
- In silicene, an external electric field can tune the band gap, so that it can behave as a normal insulator, a metal or a topological insulator.

Ezawa









- Can tune to valley and spin polarised transport by appropriate magnetic fields and barriers

  Ruchi Saxena, Arijit Saha and S. R., PRB (2015)
- Have also studied effect of shining light on these materials in the high frequency limit to obtain and understand new topological phases

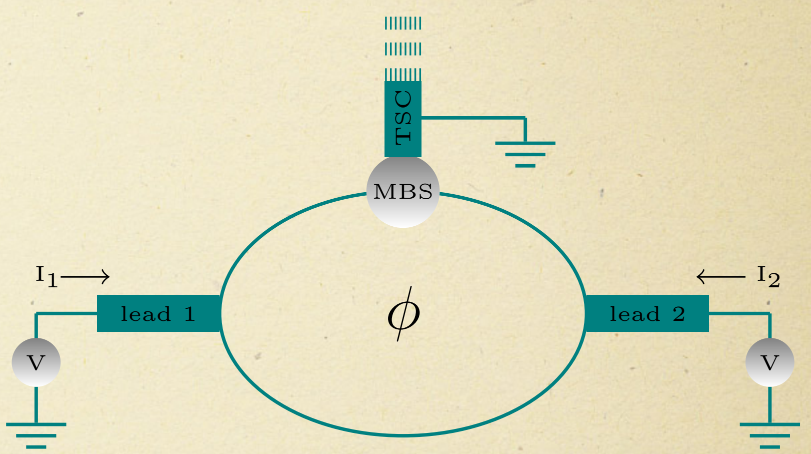
  Priyanka Mohan, Ruchi Saxena, Arijit Kundu and S.R, PRB 94, 235419, 2016
- Currently working on extending the results to lower frequencies where even more phases can be obtained and these phases are characterised by more than just the Chern numbers. Find that other winding numbers are involved.

Majorana modes

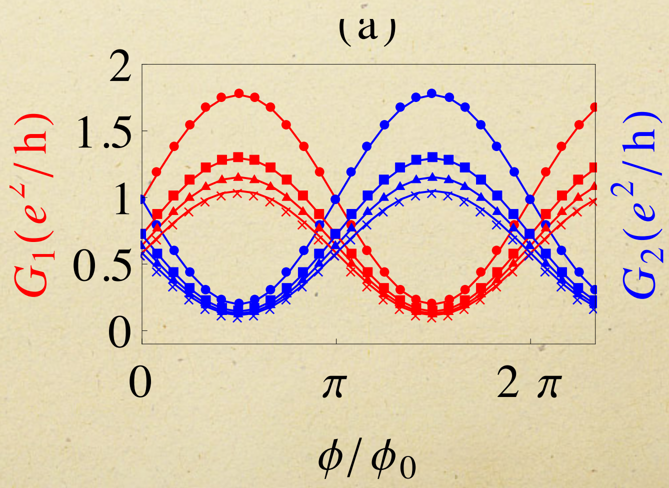
- We have also been working on Majorana modes
- The race to find Majorana modes continues
- Although several candidates exist, one problem has been the unambiguous identification of a Majorana bound state as compared to other accidental zero energy bound states
- In this context, we have proposed a ring geometry

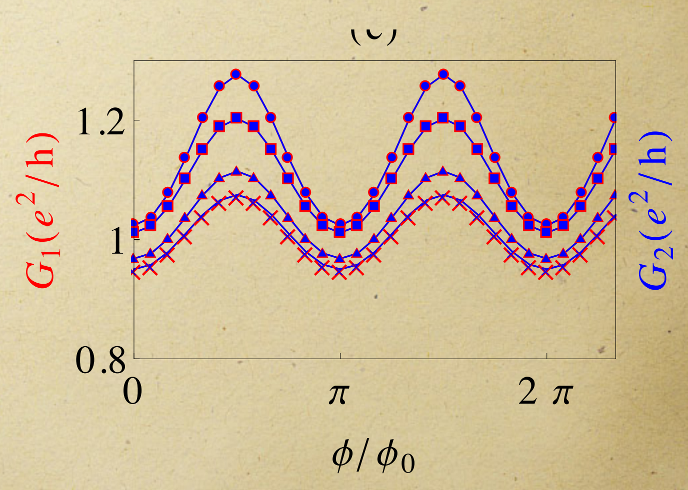
Transport through Majorana modes (non-abelian anyons)

- We have suggested that measuring currents as a function of the Aharanov-Bohm flux in a ring geometry could distinguish between Majorana bound states and zero energy Andreev bound states at the end of a topological superconductor



Krashna Mohan Tripathi, Sourin Das and S. R, PRL, 2016





To conclude…..

- Having fun working with topological phases in various guises and hope to continue ….